Package: tptest (via r-universe)

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Type Package

Title Universal Turning Point and Inflection Point Tests

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Description Performs turning point and inflection point tests for U-shaped and inverse U-shaped relationships in regression models. Implements the Sasabuchi (1980) test as extended by Lind and Mehlum (2010) with support for quadratic, cubic, log-quadratic, and inverse functional forms. Features include delta-method standard errors, Fieller confidence intervals, Simonsohn (2018) two-lines test, and parametric bootstrap. Designed for post-estimation analysis of linear models, panel models, and quantile regression. References: Lind and Mehlum (2010) <doi:10.1111/j.1468-0084.2009.00569.x>; Sasabuchi (1980); Fieller (1954) <doi:10.1111/j.2517-6161.1954.tb00159.x>.

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URL https://github.com/muhammedalkhalaf/tptest

BugReports https://github.com/muhammedalkhalaf/tptest/issues

Encoding UTF-8

LazyData true

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Depends R (>= 4.0.0)

Imports stats, graphics

Suggests testthat (>= 3.0.0), knitr, rmarkdown, lmtest, sandwich, plm, quantreg

Repository https://muhammedalkhalaf.r-universe.dev

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Contents

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ekc *Environmental Kuznets Curve Data*

Description

Simulated data representing the Environmental Kuznets Curve (EKC) hypothesis, which posits an inverse U-shaped relationship between environmental degradation and economic development.

Usage

```
ekc
```

Format

A data frame with 500 observations and 6 variables:

country Country identifier (1-50)

year Year (2000-2009)

gdp GDP per capita (thousands of dollars)

gdp_sq Squared GDP per capita

emissions CO2 emissions (tons per capita)

emissions_log Log of CO2 emissions

Details

This is simulated panel data designed to demonstrate the tptest package. The true data generating process follows an inverse U-shape with:

- Turning point at GDP ~ 25,000 USD per capita
- Country-specific fixed effects
- Year-specific trends

Value

A data frame containing Environmental Kuznets Curve data with columns for income and emissions.

Source

Simulated data for demonstration purposes.

References

Grossman, G. M., & Krueger, A. B. (1995). Economic growth and the environment. *Quarterly Journal of Economics*, 110(2), 353-377.

Examples

```
data(ekc)
head(ekc)

# Fit quadratic model
fit <- lm(emissions ~ gdp + gdp_sq, data = ekc)
summary(fit)

# Test for inverse U-shape
result <- tptest(fit, vars = c("gdp", "gdp_sq"), data = ekc)
print(result)
```

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fieller_ci *Fieller Confidence Interval for Ratio of Coefficients*

Description

Computes the Fieller (1954) confidence interval for the turning point, which is more appropriate when the denominator coefficient has high uncertainty.

Usage

```
fieller_ci(b1, b2, s11, s12, s22, level = 0.95, form = "quadratic")
```

Arguments

b1	First coefficient
b2	Second coefficient
s11	Variance of b1
s12	Covariance of b1 and b2
s22	Variance of b2
level	Confidence level
form	Functional form ("quadratic" or "inverse")

Value

A list with elements `lo`, `hi`, and `type`.

References

Fieller, E. C. (1954). Some problems in interval estimation. *Journal of the Royal Statistical Society: Series B*, 16(2), 175-185. doi:10.1111/j.25176161.1954.tb00159.x

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`tptest` *Universal Turning Point and Inflection Point Test*

Description

Tests for U-shaped or inverse U-shaped relationships using the Sasabuchi (1980) test as extended by Lind and Mehlum (2010). Supports quadratic, cubic, log-quadratic, and inverse functional forms.

Usage

```
tptest(
  model = NULL,
  vars,
  coefs = NULL,
  vcov_mat = NULL,
  min = NULL,
  max = NULL,
  form = c("auto", "quadratic", "cubic", "inverse", "logquadratic"),
  level = 0.95,
  delta = TRUE,
  fieller = FALSE,
  twolines = FALSE,
  bootstrap = FALSE,
  breps = 1000,
  data = NULL,
  depvar = NULL
)
```

Arguments

`model`	A fitted model object (e.g., from `lm`, `glm`). Alternatively, coefficients can be provided directly via `coefs`.
`vars`	Character vector of length 2 or 3 specifying the variable names: `c("x", "x_sq")` for quadratic, `c("x", "x_sq", "x_cu")` for cubic.
`coefs`	Named numeric vector of coefficients. If provided, `model` is not required. Must include names matching `vars`.
`vcov_mat`	Variance-covariance matrix for the coefficients. Required when `coefs` is provided.

`min`	Lower bound of the data interval. If `NULL`, extracted from the model data.
`max`	Upper bound of the data interval. If `NULL`, extracted from the model data.
`form`	Functional form: `"auto"` (default), `"quadratic"`, `"cubic"`, `"inverse"`, or `"logquadratic"`.
`level`	Confidence level for intervals (default 0.95).
`delta`	Logical; compute delta-method SE and CI (default `TRUE`).
`fieller`	Logical; compute Fieller confidence interval (default `FALSE`).
`twolines`	Logical; perform Simonsohn (2018) two-lines test (default `FALSE`).
`bootstrap`	Logical; compute parametric bootstrap CI (default `FALSE`).
`breps`	Number of bootstrap replications (default 1000).
`data`	Optional data frame for two-lines test.
`depvar`	Name of dependent variable for two-lines test.

Details

The function implements several approaches for testing non-monotonic relationships:

Sasabuchi (1980) / Lind-Mehlum (2010) Test: Tests whether the relationship is U-shaped (or inverse U-shaped) by examining slopes at the interval boundaries. The null hypothesis is monotonicity or opposite U-shape.

Functional Forms:

- **Quadratic:** $y = \beta_1 x + \beta_2 x^2$; turning point at $x^* = -\beta_1/(2\beta_2)$
- **Cubic:** $y = \beta_1 x + \beta_2 x^2 + \beta_3 x^3$; up to two turning points
- **Inverse:** $y = \beta_1 x + \beta_2/x$; turning point at $x^* = \sqrt{\beta_2/\beta_1}$
- **Log-quadratic:** $\ln(y) = \beta_1 \ln(x) + \beta_2[\ln(x)]^2$

Value

An object of class `"tptest"` containing:

tp Turning point estimate

tp_se Delta-method standard error

tp_ci Confidence interval for turning point

shape Detected shape ("U shape" or "Inverse U shape")

model_form Functional form used

sasabuchi List with Sasabuchi test results

fieller Fieller interval (if requested)

twolines Two-lines test results (if requested)

bootstrap Bootstrap results (if requested)

coefficients Named vector of relevant coefficients

vcov Variance-covariance matrix

bounds Data interval bounds

References

Lind, J. T., & Mehlum, H. (2010). With or without U? The appropriate test for a U-shaped relationship. *Oxford Bulletin of Economics and Statistics*, 72(1), 109-118. doi:10.1111/j.1468-0084.2009.00569.x

Sasabuchi, S. (1980). A test of a multivariate normal mean with composite hypotheses determined by linear inequalities. *Biometrika*, 67(2), 429-439.

Fieller, E. C. (1954). Some problems in interval estimation. *Journal of the Royal Statistical Society: Series B*, 16(2), 175-185. doi:10.1111/j.25176161.1954.tb00159.x

Simonsohn, U. (2018). Two lines: A valid alternative to the invalid testing of U-shaped relationships with quadratic regressions. *Advances in Methods and Practices in Psychological Science*, 1(4), 538-555.

Examples

```
# Simulate data with U-shaped relationship
set.seed(42)
n <- 200
x <- runif(n, 1, 10)
y <- 50 - 8*x + 0.5*x^2 + rnorm(n, sd = 5)
dat <- data.frame(y = y, x = x, x_sq = x^2)

# Fit quadratic model
fit <- lm(y ~ x + x_sq, data = dat)

# Test for U-shape
result <- tptest(fit, vars = c("x", "x_sq"), data = dat)
print(result)


# With Fieller interval and two-lines test
result2 <- tptest(fit, vars = c("x", "x_sq"),
                  fieller = TRUE, twolines = TRUE,
                  data = dat, depvar = "y")
summary(result2)
```

`tptest-methods` *Methods for tptest Objects*

Description

Print, summary, plot, and accessor methods for `tptest` objects.

Usage

```
## S3 method for class 'tptest'
print(x, ...)

## S3 method for class 'tptest'
summary(object, ...)

## S3 method for class 'tptest'
plot(
  x,
  main = NULL,
  xlab = "x",
  ylab = "Marginal Effect",
  col.line = "steelblue",
  col.tp = "red",
  col.ci = grDevices::rgb(0.2, 0.4, 0.8, 0.2),
  lwd = 2,
  n = 200,
  ...
)

## S3 method for class 'tptest'
coef(object, ...)

## S3 method for class 'tptest'
confint(object, parm = NULL, level = NULL, ...)
```

Arguments

x	A tptest object
...	Additional arguments (currently ignored)
object	A tptest object
main	Plot title
xlab	X-axis label
ylab	Y-axis label
col.line	Color for the fitted curve
col.tp	Color for the turning point marker
col.ci	Color for confidence band
lwd	Line width
n	Number of points for plotting curve
parm	Parameter specification (currently ignored)
level	Confidence level (default uses level from tptest object)

Value

For `print` and `summary`: the input object, returned invisibly. For `plot`: no return value, called for side effects (generates plots).

`twolines_test` *Simonsohn (2018) Two-Lines Test*

Description

Performs the two-lines test proposed by Simonsohn (2018) as an alternative to quadratic regression for testing U-shaped relationships.

Usage

```
twolines_test(data, x_var, y_var, split_point, form = "quadratic")
```

Arguments

`data`	Data frame containing the variables
`x_var`	Name of the x variable (character)
`y_var`	Name of the y variable (character)
`split_point`	Point at which to split the data (usually the turning point)
`form`	Functional form (for log-quadratic, split is in log-space)

Value

A list with test results including slopes, t-values, p-values, and whether the test confirms the U-shape.

References

Simonsohn, U. (2018). Two lines: A valid alternative to the invalid testing of U-shaped relationships with quadratic regressions. *Advances in Methods and Practices in Psychological Science*, 1(4), 538-555.

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