Package: boundedur (via r-universe)

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Type Package

Title Unit Root Tests for Bounded Time Series

Version 1.0.3

Description Implements unit root tests for bounded time series following Cavaliere and Xu (2014) <doi:10.1016/j.jeconom.2013.08.026>. Standard unit root tests (ADF, Phillips-Perron) have non-standard limiting distributions when the time series is bounded. This package provides modified ADF and M-type tests (MZ-alpha, MZ-t, MSB) with p-values computed via Monte Carlo simulation of bounded Brownian motion. Supports one-sided (lower bound only) and two-sided bounds, with automatic lag selection using the MAIC criterion of Ng and Perron (2001) <doi:10.1111/1468-0262.00256>.

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URL https://github.com/muhammedalkhalaf/boundedur

BugReports https://github.com/muhammedalkhalaf/boundedur/issues

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Depends R (>= 3.5.0)

Imports stats

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Repository https://muhammedalkhalaf.r-universe.dev

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boundedur *Unit Root Tests for Bounded Time Series*

Description

Performs unit root tests for time series constrained within known bounds, following the methodology of Cavaliere and Xu (2014). Provides modified ADF and M-type test statistics with p-values computed via Monte Carlo simulation of bounded Brownian motion.

Usage

```
boundedur(
  y,
  lbound,
  ubound = Inf,
  test = c("all", "adf", "adf_alpha", "adf_t", "mz_alpha", "mz_t", "msb"),
  lags = NULL,
  maxlag = NULL,
  detrend = c("constant", "none"),
  nsim = 499,
  nstep = NULL,
  seed = NULL
)
```

Arguments

`y`	Numeric vector. The time series to test.
`lbound`	Numeric. Lower bound for the series.
`ubound`	Numeric or `Inf`. Upper bound for the series. Use `Inf` for one-sided (lower bound only) testing.
`test`	Character. Which test(s) to perform. One of: • `"all"`: All available tests (default) • `"adf"`: Both ADF-alpha and ADF-t • `"adf_alpha"`: ADF normalized bias test only • `"adf_t"`: ADF t-statistic test only • `"mz_alpha"`: MZ-alpha test only • `"mz_t"`: MZ-t test only • `"msb"`: MSB test only

lags	Integer or NULL. Number of lagged differences to include. If NULL (default), selected automatically using MAIC.
maxlag	Integer or NULL. Maximum lag for automatic selection. If NULL, uses floor(12 * (T/100)^0.25).
detrend	Character. Detrending method: • "constant": Demean the series (default) • "none": No detrending
nsim	Integer. Number of Monte Carlo replications for p-value computation. Default is 499.
nstep	Integer or NULL. Number of discretization steps for Brownian motion simulation. If NULL, uses sample size T.
seed	Integer or NULL. Random seed for reproducibility.

Details

Standard unit root tests assume the series is unbounded, leading to non-standard limiting distributions when bounds are present. This function implements the bounded unit root tests of Cavaliere and Xu (2014), which account for the effect of bounds on the limiting distribution.

The null hypothesis is that the series has a unit root while respecting the bounds. The alternative is stationarity.

Value

An object of class "boundedur" containing:

statistics	Named vector of test statistics
p_values	Named vector of p-values
results	Data frame with statistics, p-values, and decisions
n	Sample size
lags	Number of lags used
lbound	Lower bound
ubound	Upper bound
c_lower	Standardized lower bound parameter
c_upper	Standardized upper bound parameter
sigma2_lr	Autoregressive spectral estimate $s^2_{AR}(k)$ of the long-run variance
alpha1	$\hat{\alpha}(1)$, one minus the sum of the lag coefficients
x0	Initial observation X_0 used in the bound parameters
detrend	Detrending method used
nsim	Number of Monte Carlo replications
call	The matched call

Test Statistics

ADF-alpha $T(\hat{\rho} - 1)/\hat{\alpha}(1)$ from the ADF regression of the de-meaned series (no deterministic terms)

ADF-t The t-statistic for $\rho - 1 = 0$ in that regression

MZ-alpha $(T^{-1}\hat{X}_T^2 - T^{-1}\hat{X}_0^2 - s_{AR}^2)/(2T^{-2}\sum \hat{X}_{t-1}^2)$

MZ-t $MZ_\alpha \times MSB$

MSB $(T^{-2}\sum \hat{X}_{t-1}^2/s_{AR}^2)^{1/2}$

All five reject for small values; ADF-alpha and MZ-alpha, and ADF-t and MZ-t, share the same limiting distribution (Theorem 1).

P-value Computation

P-values follow Algorithm 1 of Cavaliere and Xu (2014): the bound parameters are estimated by $\hat{c} = (b - X_0)/(s_{AR}T^{1/2})$, and the null distribution is simulated from a random walk regulated at $\hat{c}$ and $\bar{c}$. The number of replications (`nsim`) controls accuracy; larger values give more precise p-values but increase computation time.

References

Cavaliere, G., & Xu, F. (2014). Testing for unit roots in bounded time series. *Journal of Econometrics*, 178(2), 259-272. doi:10.1016/j.jeconom.2013.08.026

Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69(6), 1519-1554. doi:10.1111/14680262.00256

Examples

```
# Generate bounded random walk (interest rate between 0 and 10)
set.seed(123)
n <- 200
y <- numeric(n)
y[1] <- 5
for (i in 2:n) {
  y[i] <- y[i-1] + rnorm(1, 0, 0.5)
  y[i] <- max(0, min(10, y[i]))  # Reflect at bounds
}

# Test for unit root with known bounds
result <- boundedur(y, lbound = 0, ubound = 10, nsim = 199)
print(result)
summary(result)

# One-sided bound (e.g., price level, lower bound = 0)
result_lower <- boundedur(y, lbound = 0, ubound = Inf, nsim = 199)
```

`select_lag_maic` *Select Optimal Lag using MAIC Criterion*

Description

Selects the number of lagged differences in the ADF regression with the Modified Akaike Information Criterion (MAIC) of Ng and Perron (2001).

Usage

```
select_lag_maic(y, maxlag = NULL, detrend = "constant")
```

Arguments

`y`	Numeric vector. Time series data.
`maxlag`	Integer or `NULL`. Maximum lag to consider. If `NULL`, uses the rule `floor(12 * (T/100)^0.25)`.
`detrend`	Character. De-meaning method: "constant" (OLS de-meaning) or "none". Default is "constant".

Details

For $k = 0, \ldots, k_{max}$, the ADF regression of $\Delta \hat{X}_t$ on $\hat{X}_{t-1}$ and k lagged differences is estimated on the common sample $t = k_{max} + 2, \ldots, T$, and

$$MAIC(k) = \ln(\hat{\sigma}_k^2) + 2(\tau_T(k) + k)/(T - k_{max} - 1),$$

with $\tau_T(k) = \hat{\sigma}_k^{-2} \hat{\beta}_0^2 \sum \hat{X}_{t-1}^2$ and $\hat{\beta}_0$ the coefficient on $\hat{X}_{t-1}$.

Value

A list with class `"lag_selection"` containing:

`selected_lag`	Optimal lag selected by MAIC
`maic`	MAIC value at optimal lag
`all_maic`	Vector of MAIC values for all lags
`maxlag`	Maximum lag considered
`n`	Sample size

References

Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, 69(6), 1519-1554. doi:10.1111/14680262.00256

Examples

```
# Generate random walk
set.seed(123)
y <- cumsum(rnorm(200))

# Select lag
lag_sel <- select_lag_maic(y)
print(lag_sel)
```

`simulate_bounded_bm` *Simulate Bounded Brownian Motion*

Description

Simulates the discretized regulated Brownian motion of Algorithm 1 in Cavaliere and Xu (2014).

Usage

```
simulate_bounded_bm(n, c_lower, c_upper = Inf)
```

Arguments

`n`	Integer. Number of time steps for discretization.
`c_lower`	Numeric. Standardized lower bound parameter.
`c_upper`	Numeric or `Inf`. Standardized upper bound parameter. Use `Inf` for one-sided (lower) bound only.

Details

The path follows the recursion (4.11) of Cavaliere and Xu (2014): $X_t = X_{t-1} + n^{-1/2}\varepsilon_t$, set to the bound whenever it would cross it, with $X_0 = 0$ and i.i.d. standard normal ε_t.

The standardized bound parameters `c_lower` and `c_upper` are computed from the original bounds as:

$$c = (b - X_0)/(\sigma\sqrt{T})$$

where b is the bound, X_0 is the initial value, σ is the long-run standard deviation, and T is the sample size.

Value

A numeric vector of length `n + 1` containing the simulated bounded Brownian motion path, starting at 0.

References

Cavaliere, G., & Xu, F. (2014). Testing for unit roots in bounded time series. *Journal of Econometrics*, 178(2), 259-272. doi:10.1016/j.jeconom.2013.08.026

Examples

```
# Simulate bounded Brownian motion with two-sided bounds
set.seed(123)
bm <- simulate_bounded_bm(n = 1000, c_lower = -2, c_upper = 2)
plot(bm, type = "l", main = "Bounded Brownian Motion")
abline(h = c(-2, 2), col = "red", lty = 2)

# One-sided bound (lower only)
bm_lower <- simulate_bounded_bm(n = 1000, c_lower = -1, c_upper = Inf)
```

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